

Equilibrium configurations for quark-diquark stars
and the problem of Her X-1 mass

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Abstract

We report new calculations of the physical properties of a quark-diquark plasma. A vacuum contribution is taken into account and is responsible for the appearance of a stable state at zero pressure and at a baryon density of about 2.2 times the nuclear matter density in this model. The resulting equation of state was used to integrate numerically the Tolman-Oppenheimer-Volkoff equations. The mass-radius relationship has been derived from a series of equilibrium configurations constituted by a mixture of quarks and diquarks. These stellar models, which are representative of a whole class, may be helpful to understand the possible compactness of the X-ray source Her X-1 and related objects.

1.Introduction

The main static properties of neutron stars can be studied in the mass-radius (hereafter $M - R$) plane. As is well known, curves in such a diagram depend on the adopted equation of state (EOS). Generally speaking, for a given EOS in which the matter does not have a self-bound state, the radius of an equilibrium configuration decreases as the mass increases until a maximum (mass) is reached (see, for instance, Ref. 1). Objects less massive and more compact satisfying the condition $dM/dR > 0$ are unstable.

From an observational point of view, the analysis of a given object in the $M - R$ plane requires the determination of both quantities. Even though masses can be obtained for neutron stars in binary systems, the radius is a quantity of difficult direct access. Some constraints in the M/R ratio have been established, for example, from the analysis of a sample of X-ray bursters ².

More recently, Li, Dai and Wang³ have examined the location of the binary X-ray pulsar Her X-1 in the plane $M - R$. The mass ($0.98 \pm 0.12 M_{\odot}$) was calculated using the orbital parameters of the system, whereas the radius of the pulsar (6.7 ± 1.2 km) was estimated using the accretion torque model^{4,5}. The compactness of Her X-1 was previously considered by Mészáros and Riffert⁶, who obtained an even smaller value for the radius (5-6 km) using similar arguments. In spite of the indirect nature of this estimate, it is seems interesting to investigate the consequences when these observational quantities are compared with theoretical expectations. In fact, as Li,Dai and Wang noticed, the majority of the existing EOS predict a considerably higher radius, for a stable configuration of that mass. In particular, pure hadron stars (see Ref.1) or models including a quark core, (see

Ref. 7 and references therein) predict a radius of about 11.5 km for a star with the same mass as Her X-1. Thus, if the accretion torque and the cyclotron line formation theories are correct, there is an indication that Her X-1 could be more compact than the configurations expected from models based on the present known EOS.

In their recent work Li, Dai and Wang further suggested that Her X-1 could be a strange star, since these objects could be more compact than neutron stars for certain parametrised QCD quantities, and thus more consistent with these derived properties. The stability of strange matter and strange star models using a simple EOS were considered first by Witten⁸ and Farhi and Jaffe⁹. More detailed models and their physical properties were investigated by Haensel et al.¹⁰ and Alcock et al.¹¹, whereas their mechanical properties were explored by Benvenuto and Horvath¹². However, as pointed out by Madsen¹³ the value of the most important QCD parameter (namely the vacuum energy B_{bag}) needed for a compact strange star is not consistent with the postulated self-boundness of this phase. Therefore, strange matter is not likely to provide an acceptable model for Her X-1.

Generally speaking, a plasma of quarks and gluons is expected to form at some level as a consequence of the disassembling of neutrons in stellar matter at densities exceeding that of the nuclear matter (0.16 fm^{-3}). However, it may be possible that the deconfinement of hadrons occurs through an intermediate stage, in which nucleons are dissociated but quarks are correlated in spin-singlet pairs, known as diquarks (for a review of diquark properties, see Ref.14). Since the suggestion by Donoghue and Sateesh¹⁵ that neutron matter may dissolve into a *highly correlated quark fluid*, several papers were devoted to discuss the physical properties of such a state. Horvath et al. (Ref. 16, hereafter paper I) calculated

the properties of a diquark-quark mixture, in which diquarks interact through a boson field described by a " $\lambda\phi^4$ " effective potential. Kastor and Traschen¹⁷ have previously computed a series of stellar models and their corresponding $M - R$ relationship. However, because of the inclusion of an external low-density neutron envelope below a threshold density, their models are not suitable to explain the compactness of Her X-1 since they behave akin to the hybrid stars⁷.

In the present work we revisited the EOS of a diquark-quark mixture, including the vacuum contribution. The vacuum energy density introduces a negative pressure able to produce an absolute stable state¹⁸, and open the possibility for the existence of a self-bound pure quark-diquark star. We report in this paper calculations of relativistic star configurations and discuss how they relate to the Her X-1 compactness problem.

2. The Equation of State

2.1. Diquarks

In order to calculate the EOS for a quark-diquark mixture, we consider the spin-dependent colormagnetic interaction leading to a spin-0 state, whose Hamiltonian is given in paper I. Diquarks of mass m_D are assumed to interact through a $\lambda\phi^4$ effective potential due to a bosonic field, and precludes a Bose condensate. The coupling parameter λ may be estimated from the Δ -N mass difference and a variant of the P-matrix formalism¹⁹. Since the coupling constant λ is found to be several orders of magnitude larger than the critical value $\lambda_* = \frac{4\pi m_D^2}{m_{Plank}^2} \approx 3 \times 10^{-36}$, we are led to use the results of Colpi et al.²⁰, who obtained parametric solutions for the EOS of particles interacting through such an effective potential. In this case, the pressure and the energy density due to diquark-diquark

interactions are respectively

$$P_D = \varepsilon_0(z - 1)^2 \quad (1)$$

and

$$\varepsilon_D = \varepsilon_0(z - 1)(3z + 1) \quad (2)$$

where $z \geq 1$ is a dimensionless parameter and ε_0 is an energy-density scale. If we adopt the same values as in paper I, namely $m_D = 575 \text{ MeV}$ for the diquark mass and $\lambda = 27.8$ for the coupling constant, the energy density scale is found to be $\varepsilon_0 = 32.1 \text{ MeV fm}^{-3}$.

2.2. Quarks

Due to the broken chiral symmetry just after deconfinement, quarks acquire a mass near the constituent value, i.e., $m_q \approx 360 \text{ MeV}$, which we assume to be the same for both flavors u, d . If we further assume that the free quarks constitute a Fermi fluid, the energy density for a given flavor is

$$\varepsilon_{u,d} = \frac{3}{10} \frac{\pi^{4/3} \hbar^2}{m_q} n_{u,d}^{5/3} \quad (3)$$

while the pressure is given by

$$P_{u,d} = \frac{1}{5} \frac{\pi^{4/3} \hbar^2}{m_q} n_{u,d}^{5/3} \quad (4)$$

where $n_{u,d}$ stands for the number density of quarks u or d .

2.3. The quark-diquark mixture

The properties of the quark-diquark plasma are calculated neglecting the contribution from a leptonic component (electrons and muons) because they are not needed to neutralize the bulk matter. In this case, the first condition to be satisfied is the electrical charge neutrality, i.e.,

$$\frac{1}{3}n_D + \frac{2}{3}n_u - \frac{1}{3}n_d = 0 \quad (5)$$

The second condition states the conservation of the baryon charge, namely,

$$n_B = \frac{2}{3}n_D + \frac{1}{3}(n_u + n_d) \quad (6)$$

where n_B is the total baryon number density.

The total pressure of the mixture is

$$P = P_D + P_u + P_d - B \quad (7)$$

where B is the energy density of the vacuum, which in our computations it is assumed to be equal to 57 MeV fm^{-3} . We note that this choice is made without assuming that B must be the same as the original MIT bag, since there are no compelling reason to force such an identification (see below). Finally, the total energy density is given by

$$\varepsilon = m_q c^2 (n_u + n_d) + \varepsilon_D + \varepsilon_u + \varepsilon_d + B$$

where the first term on the right represents the rest energy density of the quarks.

In order to obtain the equilibrium number density of each constituent, the total energy density was minimized with respect to n_D , keeping n_B constant. In fact, the condition $(\partial\varepsilon/\partial n_D)_{n_B} = 0$ is equivalent to impose the equality between the chemical potential of diquarks and the sum of the chemical potential of the two quark flavors.

The system of equations above was solved numerically. For a given value of the total baryon density n_B , we have computed the number density and the pressure contribution of each constituent, as well as the total pressure and the total energy density. The results of our calculations are given in Table 1. In the first four columns, we give respectively (in fm^{-3}) the number density of baryons, diquarks and of the u and d quark flavors. In

the three following columns, we give the pressure (in $GeV\ fm^{-3}$) contributed by diquarks, by the sum of u and d flavours (P_{u+d}) and the total pressure, including the vacuum contribution. The last column gives the total energy density of the mixture (in $GeV\ fm^{-3}$). From these results we note that diquarks carry, on the average, about 60% of the baryonic charge. At high densities, diquarks contribute to about a half of the total pressure, and that fraction increases as the density decreases. Most of the u quarks are paired with d 's and thus their contribution to the pressure and energy density is quite small. For baryon densities smaller than $0.6\ fm^{-3}$, all u 's are paired and the diquark number density is equal to that of d quarks. A similar conclusion was also reached by Kastor and Traschen¹⁷. The sound velocity in such a fluid can be obtained from the relation $v_s = c(\partial P/\partial \epsilon)^{1/2}$. From our results, $\frac{v_s}{c} \approx 0.55$, indicating no violation of causality. An important point to be emphasized is that at a baryon density of $0.36\ fm^{-3}$ (about 2.2 times the nuclear density), the total pressure of the mixture is zero, since the adopted vacuum pressure compensates the contribution due to d quarks and diquarks. In these conditions, our calculations indicate that the energy per baryon number unit is about $834\ MeV$, smaller than the nucleon rest energy. This corresponds to a stable state (see Ref. 18 for a discussion) allowing the existence of a pure quark-diquark star. We note also that the energy density at $P = 0$ is about $7\ B$, whereas for a pure quark plasma such a value for the energy density is $4\ B$. A plot of the equation of state is given in Fig.1.

3.Stellar Models

The equation of state obtained for the quark-diquark mixture and the well known Tolman-Oppenheimer-Volkoff (TOV) equations were integrated numerically, following the

same procedure as Freitas Pacheco et al.⁷. For a given baryonic central density, the TOV equations were integrated outwards until the condition $P = 0$ is satisfied.

The resulting $M - R$ relationship obtained from these calculations is shown in Fig.2, while the plot of $n_B - M$ is shown in Figure 3. Stars of low mass ($M < 0.8M_\odot$) are nearly uniform, since the density ratio between the center and the surface is less than a factor of two. The maximum mass for a stable configuration is $1.18 M_\odot$, corresponding to a radius of 7.35 km and a central density of $1.1 fm^{-3}$, or about 6.6 times the density of nuclear matter. In comparison with pure quark stars, diquark-quark stars are more compact, and this may be partially attributed to the role played by the (poorly known) confinement forces represented by the vacuum constant B and the very presence of diquarks.

It is interesting to compare with the $M - R$ relation for strange stars^{8,10,11,12} for the same value of the vacuum constant $57 MeV fm^{-3}$. Those strange star models are not able to provide the $M - R$ values needed for Her X-1. In fact, Li et al. obtain a compatibility only for strange stars models in which the vacuum energy density is in the range 120-200 $MeV fm^{-3}$, considerably higher than the MIT scale and certainly inconsistent with the strange matter hypothesis⁹. It is only by imposing those high values of B that the required compactness is obtained¹³. In our models the quark-diquark mixture produces a softer EOS than a pure quark plasma and, as a consequence, a more compact equilibrium configuration for the same mass and vacuum energy density.

If Her X-1 is really quite compact, than a stable pure quark-diquark star is compatible with the required $M - R$ values, without forcing the acceptable range of physical parameters like the vacuum energy density or the particle effective masses. However the maximum

stable mass for these particular models is slightly lower than the derived value of the more massive member in the binary pulsar PSR 1913+16 ($1.44 M_{\odot}$). It's worth mentioning that other X-ray sources other than Her X-1 are suspect of being very compact. If the 4.1 keV line observed in the burster MXB 1636-536 is a gravitationally redshifted $K\alpha Fe$ line²¹, then the use of a "canonical" neutron star forces a radius of only 6.7 km. However, the inclusion of the transverse Doppler effect due to rotation of the accreted gas²² may increase the radius up to 10 km, compatible with neutron stars modeled through some popular EOS. Fujimoto and Gottwald²³ based on the assumption that the burster MXB 1728-34 attained the Eddington limit at maximum flux, and modeling the X-ray "color" evolution, concluded that the compact object must have a mass of $M \approx 1.2M_{\odot}$ and a radius of $R \approx 7.5km$. These values match those we derived for a diquark-quark star near the instability limit. It is important to note that at least three independent methods have been employed to determine these low-radius figures.

4. Conclusions

We have reported new calculations of the physical conditions prevailing in a mixture of deconfined quarks and diquarks, satisfying the condition of zero charge. The inclusion of confining forces (vacuum) induces the existence of a stable state of the mixture at $P = 0$, and at a baryon density of about 2.2 times the nuclear matter density. This state has an energy density of about $7 B$, which should be compared to the value $4 B$ derived for stable strange matter⁸. Moreover, it is clear that this property is shared by a family of EOS of the same type parametrised by the vacuum energy B . Although we have only given our results for a fixed B numerically equal to the MIT bag constant, this quantity should be

considered as a parameter measuring of the uncertain behaviour of the QCD vacuum and need *not* be related to the former.

The TOV equations were solved numerically, and the resulting equilibrium configurations are more compact than strange stars. In particular, a quark-diquark star is able to explain the $M - R$ values of Her X-1, if we accept that the radius is correctly estimated from the accretion torque theory.

The softness of the resulting EOS for the quark-diquark mixture implies in a maximum stable mass of only $1.18 M_{\odot}$, and thus the presented models would not be able to explain the nature of other compact stars like the more massive component of the binary pulsar PSR 1913+16. We have emphasized that there is some evidence that other objects like MXB 1728-34 may also be quite compact. In spite of the fact that the mass and the radius are estimated from simple models, (and therefore subject to some change) the compactness of these X-ray sources certainly raises some problems for which these exotic models may be useful. The main reason is that $M \rightarrow 0$ when $R \rightarrow 0$ for stars of self-bound matter and the models are thus "naturally" compact for relatively low masses. It is clear that if the orthodox interpretation in terms of nothing but one type of neutron star is maintained, then a softer EOS is required in order to produce the necessary compactness, but probably we have to face the problem of a theoretical reduction of the maximum mass for the last stable object and conflict with the binary pulsar mass determination. The main point made in our work has been to show the existence of a whole class of exotic models for which we state that a subset of them²⁴ can satisfy both the binary pulsar mass bound and the compactness of Her X-1 and related sources. Alternatively, the evidence could

be taken as a hint for the existence of at least two types of compact stars, but if this is the case there would be no compelling need to accomodate both classes in a single stellar sequence.

Table 1

n_B	n_D	n_u	n_d	P_D	$P_{(u+d)}$	P	ε
fm^{-3}	fm^{-3}	fm^{-3}	fm^{-3}	$GeV fm^{-3}$	$GeV fm^{-3}$	$GeV fm^{-3}$	$GeV fm^{-3}$
0.36	0.36	0.00	0.36	0.039	0.018	0.000	0.40
0.40	0.40	0.00	0.40	0.046	0.021	0.010	0.45
0.48	0.48	0.00	0.48	0.063	0.030	0.036	0.55
0.60	0.57	0.03	0.63	0.082	0.047	0.072	0.71
0.84	0.77	0.07	0.91	0.128	0.086	0.157	1.05
1.16	1.04	0.12	1.29	0.200	0.154	0.298	1.56
1.51	1.34	0.17	1.68	0.289	0.243	0.475	2.15
1.88	1.66	0.22	2.10	0.393	0.351	0.687	2.82

Table 1 caption: Physical properties of an equilibrium quark-diquark mixture. Units appear as indicated, see text for details and Fig.1.

Figure captions

Figure 1. Equation of state of the quark-diquark matter (solid line). The partial pressures contributed by the diquarks P_D (long dashed line) and quarks P_{u+d} (short dashed line) are also shown.

Figure 2. $M - R$ plot for quark-diquark stars. Her X-1 mass limits are shown with dashed lines. The models indicate a radius of ~ 7 km in agreement with the analysis by Li et al. (Ref.3).

Figure 3. $n_B - M$ plot for quark-diquark stars. For the Her X-1 mass range the models have a central density of about six times the nuclear matter value.

5.References

- 1) D.Arnett and R.L.Bowers, *Astrophys.J.Supp.***33**, 415 (1977).
- 2) H.Inoue, in *The Origin and Evolution of Neutron Stars, IAU Symposium 125*, eds. D.J.Helfand and J.-H. Huang (Reidel, Dodrecht,1986) p.233.
- 3) X.-D.Li, Z.-G.Lai and Z.-R.Wang, *Astro.Astrophys.***303**, L1 (1995).
- 4) I.Wasserman and S.L.Shapiro, *Astrophys.J.***265**, 1036 (1983).
- 5) X.-D.Li and Z.-R.Wang, *Astrophys.Space Sci.***225**, 47 (1995) ; X.D.Li and Z.R.Wang, *Astrophys.Space Sci.***234**, 39 (1995).
- 6) P.Mézáros and H.Riffert, *Astrophys.J.***323**, L127 (1987).
- 7) J.A.de Freitas Pacheco, J.E.Horvath, J.C.N.de Araújo and M.Cattani, *Mon.Not.R.A.S.***260**, 499 (1993).
- 8) E.Witten, *Phys.Rev.D***30**, 272 (1984).
- 9) E.Farhi and R.L.Jaffe, *Phys.Rev.D***30**, 2379 (1984).
- 10) P.Haensel, J.L.Zdunik and R.Schaeffer, *Astron.Astrophys.***160**, 121 (1986).
- 11) C.Alcock, E.Farhi and A.V.Olinto, *Astrophys.J.***310**, 261 (1986).
- 12) O.G.Benvenuto and J.E.Horvath, *Mon.Not.R.A.S.***250**, 679 (1991).
- 13) J.Madsen, astro-ph 9601129 preprint (1996).
- 14) M.Anselmino, E.Predazzi, S.Ekelin, S.Fredriksson and D.B.Lichtenberg, *Rev.Mod.Phys.***65**, 1199 (1993).
- 15) J.Donoghue and K.S.Sateesh, *Phys.Rev.D***38**, 360 (1989).
- 16) J.E.Horvath, J.A.de Freitas Pacheco and J.C.N.de Araújo, *Phys.Rev.D* **46**, 4754 (1992), paper I.

- 17) D.Kastor and J.Traschen, *Phys.Rev.D***44**, 3791 (1991).
- 18) J.E.Horvath, *Phys.Lett.B***294**, 412 (1992).
- 19) R.L.Jaffe and F.E.Low, *Phys.Rev.D***19**, 2105 (1979).
- 20) M.Colpi, S.L.Shapiro and I.Wasserman, *Phys.Rev.Lett.***57**, 2485 (1986).
- 21) I.Waki et al., *PASJ***36**, 819 (1984).
- 22) M.Y.Fujimoto, *Astrophys.J.***293**, L13 (1985).
- 23) M.Y.Fujimoto and M.Gottwald, *Mon.Not.R.A.S***236**, 545 (1989).
- 24) J.E.Horvath and J.A.de Freitas Pacheco, in preparation.